Technical Comments

Comment on "Torque on a Satellite Due to Gravity Gradient and Centrifugal Force"

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CARROLL[1] states that, in addition to gravity-gradient torque acting on a satellite, there is a centrifugal torque, which, for a symmetric satellite in a circular orbit, has magnitude

$$T_c = (\tfrac{1}{2})\omega_0^2(I_s - I_t)\sin 2\beta'' \qquad (1)$$

He then includes this centrifugal torque in the inertial space analysis of the precession of the spin axis of a spinning satellite. This is not correct.

The centrifugal torque given by Carroll is part of the "inertial" torque which has been applied by some authors[2] in the sense of d'Alambert in order to analyze dynamics in the *gravity-oriented* frame, i.e., a frame of reference rotating at orbital angular velocity.

References

[1] Carroll, P. S., "Torque on a satellite due to gravity gradient and centrifugal force," AIAA J. 2, 2220–2222 (1964).

[2] Etkin, B., "Dynamics of gravity-oriented orbiting systems with application to passive stabilization," AIAA J. 2, 1008–1014 (1964).

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IN the Technical Note quoted,[1] the average total torque value per orbit for fixed orientation of a satellite is determined. Investigations of the same problem using Lagrangian formalism have shown a different result, which will be interpreted at the end of this contribution. First the Lagrangian formalism will be developed. For simplicity the satellite will be considered as a dumbbell. The Lagrangian of the satellite in the gravitational field of the earth will be

$$L = \tfrac{1}{2}m\, r^2\omega^2 + \tfrac{1}{2}A(\dot\alpha + \omega)^2\cos^2\beta + \tfrac{1}{2}A\dot\beta^2 + mr^2\omega^2 + \tfrac{3}{2}A\omega^2\cos^2\alpha\cos^2\beta - \tfrac{1}{2}A\omega^2$$

(circular orbit assumed)

where (Fig. 1) φ, r are the cylindrical coordinates of the dumbbell's center of mass; m, A is the mass or principal moment of inertia of the dumbbell; ω is orbital revolution rate; α is the angle from the local vertical to the projection of the dumbbell into the plane of the orbit; and β is the angle from the symmetry axis to the plane of the orbit.

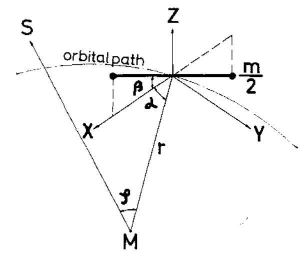

Fig. 1 Coordinate system (the drawing plane contains the x-y axes, the earth's center M, the space-fixed direction S, and the orbital path).

The equations of motion, using the Lagrangian equations follow:

$$A\ddot\alpha\cos^2\beta - A(\dot\alpha + \omega)\sin 2\beta\dot\beta + \tfrac{3}{2}\omega^2 A\sin 2\alpha\cos^2\beta = 0$$

$$A\ddot\beta + \tfrac{3}{2}\omega^2 A\sin 2\beta\cos^2\alpha + \tfrac{1}{2}(\dot\alpha + \omega)^2 A\sin 2\beta = 0$$

If there are artificial forces without potential, the Lagrange equations are not complete. In our case, we must introduce the torques caused by these forces that may be acting in the orbital plane or vertically to the plane and that will be called T_α and T_β. Then we obtain

$$A\ddot\alpha\cos^2\beta - A(\dot\alpha + \omega)\sin 2\beta\dot\beta + \tfrac{3}{2}\omega^2 A\sin 2\alpha\cos^2\beta - T_\alpha\cos\beta = 0$$

$$A\ddot\beta + \tfrac{3}{2}\omega^2 A\sin 2\beta\cos^2\alpha + \tfrac{1}{2}(\dot\alpha + \omega)^2 A\sin 2\beta - T_\beta = 0$$

In the case of fixed orientation, we have $\dot\beta = 0$ and $\dot\alpha = -\omega$, leading to

$$T_\alpha = \tfrac{3}{2}\omega^2 A\sin 2\alpha\cos\beta \qquad T_\beta = \tfrac{3}{2}\omega^2 A\sin 2\beta\cos^2\alpha$$

Averaging these torques for one revolution results in

$$T_{\alpha,\mathrm{av}} = 0 \qquad T_{\beta,\mathrm{av}} = \tfrac{3}{4}\omega^2 A\sin 2\beta$$

It is interesting to note that the same result would be obtained by averaging only the torque caused by gravity gradient. For a complete description in a rotating system, we also have to consider torques caused by centrifugal forces, torques caused by coriolis forces, and torques caused by the relative acceleration in the rotating system as well as the angular acceleration on the rotating system. This leads to the result stated previously.

There is another way to verify the result obtained: The two masses of the space-fixed dumbbell may be considered as moving in two circles with the same radius but with different centers. Then we have to consider two moving systems causing only centrifugal forces. Under this aspect, the centrifugal forces for the two masses have the same direction and the same magnitude, thus producing no torque on the satellite.

Reference

[1] Carroll, P. S., "Torque on a satellite due to gravity gradient and centrifugal force," AIAA J. 2, 2220–2222 (1964).

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